Polynomial Functions cont.

- The general form of a polynomial function is

$$s = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + \cdots C_n x^n$$

"s" represents the follower displacement and x is the independent variable which can be a linear position, $\frac{\theta}{\beta}$, or time.

We can represent a cam profile by using a polynomial function. The number of terms in the polynomial will be equal to the number of boundary conditions.

- For the double-dwell problem we can define six boundary conditions for the rise of the cam motion.

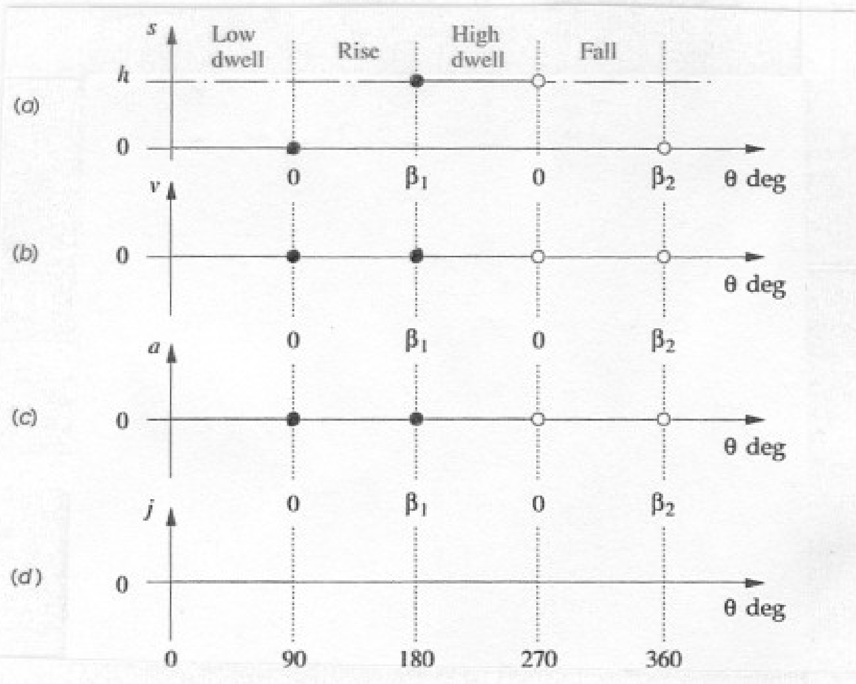

FIGURE 8-24
Minimum boundary conditions for the double-dwell case

$$s(0) = v(0) = a(0) = 0$$

$$s(\beta_1) = h$$

$$v(\beta_1) = a(\beta_1) = 0$$

- Since we have 6 boundary conditions we need a polynomial with 6 coefficients

$$s = C_0 + C_1\left(\frac{\theta}{\beta_1}\right) + C_2\left(\frac{\theta}{\beta_1}\right)^2 + C_3\left(\frac{\theta}{\beta_1}\right)^3 + C_4\left(\frac{\theta}{\beta_1}\right)^4 + C_5\left(\frac{\theta}{\beta_1}\right)^5$$

we can differentiate wrt. to θ to get v and a →

$$v = \frac{1}{\beta_1}\left[C_1 + 2C_2\left(\frac{\theta}{\beta_1}\right) + 3C_3\left(\frac{\theta}{\beta_1}\right)^2 + 4C_4\left(\frac{\theta}{\beta_1}\right)^3 + 5C_5\left(\frac{\theta}{\beta_1}\right)^4\right]$$

$$a = \frac{1}{\beta_1^2}\left[2C_2 + 6C_3\left(\frac{\theta}{\beta_1}\right) + 12C_4\left(\frac{\theta}{\beta_1}\right)^2 + 20C_5\left(\frac{\theta}{\beta_1}\right)^3\right]$$

Now we can apply the BC's to solve for $C_0, C_1, C_2, C_3, C_4, C_5$

① $s(0) = 0 = C_0 + 0 + 0 + 0 + 0 + 0 \rightarrow C_0 = 0$

② $v(0) = 0 = \frac{1}{\beta}\left[C_1 + 0 + 0 + 0 + 0\right] \rightarrow C_1 = 0$

③ $a(0) = 0 = \frac{1}{\beta^2}\left[2C_2 + 0 + 0 + 0\right] \rightarrow C_2 = 0$

④ $S(\beta_1) = h = C_3\left(\frac{\beta_1}{\beta_1}\right)^2 + C_4\left(\frac{\beta_1}{\beta_1}\right)^2 + C_5\left(\frac{\beta_1}{\beta_1}\right)^3$

$\therefore \quad h = C_3 + C_4 + C_5$

⑤ $v(\beta_1) = 0 = \frac{1}{\beta_1}\left[3C_3\left(\frac{\beta_1}{\beta_1}\right)^2 + 4C_4\left(\frac{\beta_1}{\beta_1}\right)^3 + 5C_5\left(\frac{\beta_1}{\beta_1}\right)^4\right]$

$\therefore \quad 3C_3 + 4C_4 + 5C_5 = 0$

⑥ $a(\beta_1) = 0 = \frac{1}{\beta_1^2}\left[6C_3\left(\frac{\beta_1}{\beta_1}\right) + 12C_4\left(\frac{\beta_1}{\beta_1}\right)^2 + 20C_5\left(\frac{\beta_1}{\beta_1}\right)^3\right]$

$\therefore \quad 6C_3 + 12C_4 + 20C_5 = 0$

We now have three equations and three unknowns and we can solve for the unknown coefficients

$C_3 = 10h \qquad C_4 = -15h \qquad C_5 = 6h$

substituting into the displacment function we obtain,

$$S = h\left[10\left(\frac{\theta}{\beta_1}\right)^3 - 15\left(\frac{\theta}{\beta_1}\right)^4 + 6\left(\frac{\theta}{\beta_1}\right)^5\right]$$

This is called a 3-4-5 polynomial because of the exponents

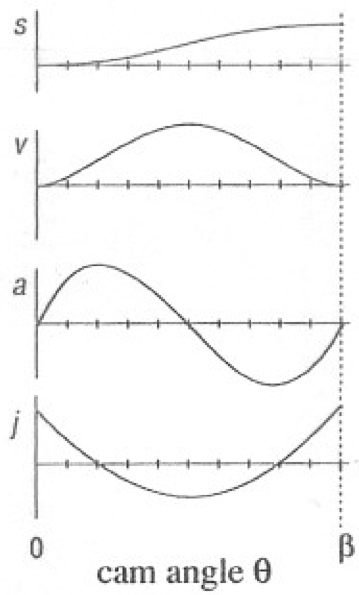

FIGURE 8-25

3-4-5 polynomial rise. Its acceleration is very similar to the sinusoid of cycloidal motion

- For the 3-4-5 polynomial rise function velocity and acceleration are continuous but jerk is not. That's because jerk was left unconstrained. In order to constrain the jerk we need to add two additional coefficients C_6 and C_7 and use two more boundary conditions

$$j(0) = j(\beta_1) = 0$$

- If we repeat the same analysis but add the two extra BC's and coefficients, we will result in the following 4-5-6-7 polynomial for the displacement function

$$S = h\left[35\left(\frac{\theta}{\beta_1}\right)^4 - 84\left(\frac{\theta}{\beta_1}\right)^5 + 70\left(\frac{\theta}{\beta_1}\right)^6 - 20\left(\frac{\theta}{\beta_1}\right)^7\right]$$

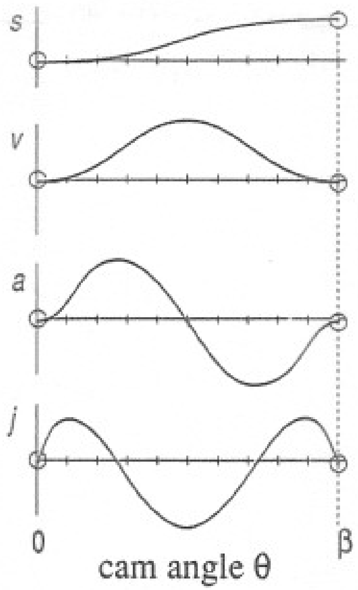

FIGURE 8-26

4-5-6-7 polynomial rise. Its jerk is piecewise continuous with the dwells

- This 4-5-6-7 polynomial will have a smoother jerk but a higher acceleration than the 3-4-5 polynomial. See pp 4-2 and 4-3 for the graphical comparison.

Single Dwell Cams

Many applications require a single dwell cam profile. For example the cam that opens valves in an engine. The valve opens on the rise, closes on the return, and remains shut while combustion and compression takes place. The cam profiles for the double dwell case may work but will not be optimal.

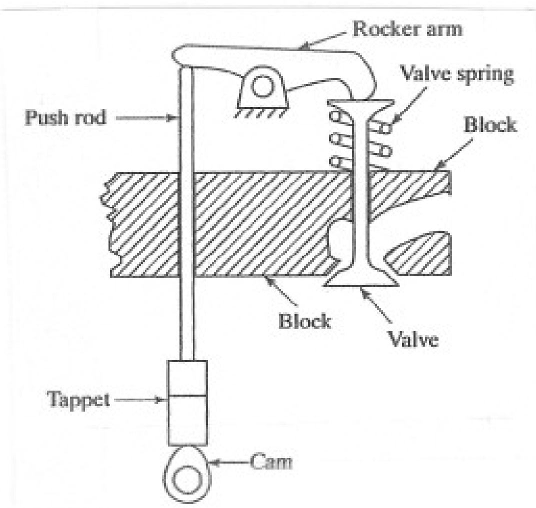

FIGURE 5.26 Valve train schematic (not to scale).

(see pp 2-1)

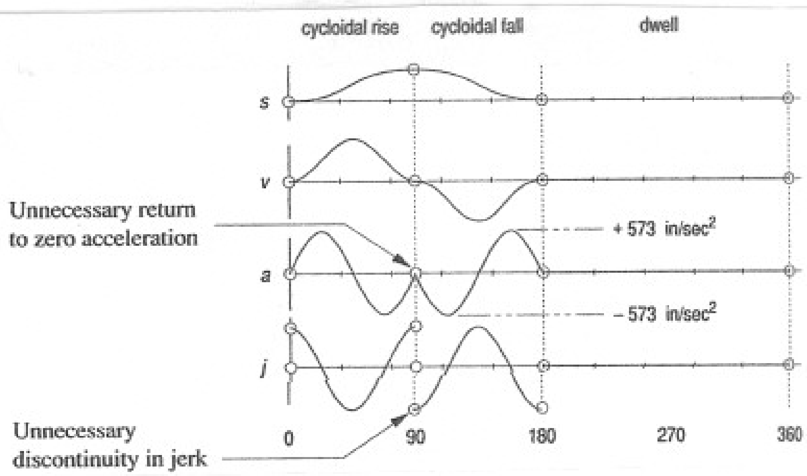

FIGURE 8-27

Cycloidal motion (or any double-dwell program) is a poor choice for the single-dwell case

- Although the jerk is finite, it is discontinuous. A better approach is to use the double harmonic function

$$S = \frac{h}{2}\left\{\left[1 - \cos\left(\pi \frac{\theta}{\beta}\right)\right] - \frac{1}{4}\left[1 - \cos\left(2\pi \frac{\theta}{\beta}\right)\right]\right\} \quad \text{(Rise)}$$

$$S = \frac{h}{2}\left\{\left[1 + \cos\left(\frac{\pi\theta}{\beta}\right)\right] - \frac{1}{4}\left[1 - \cos\left(2\pi \frac{\theta}{\beta}\right)\right]\right\} \quad \text{(Return)}$$

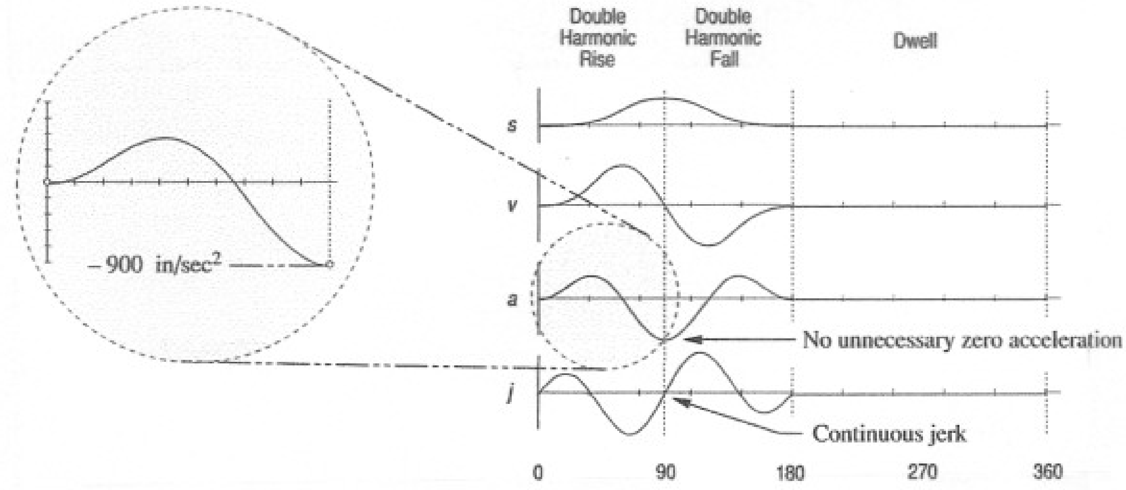

FIGURE 8-28
Double harmonic motion can be used for the single-dwell case if rise and fall durations are equal

- If we used a polynomial function to design the single dwell cam, we could use only one function to describe the rise and the return. The BC's are:

$$s(0) = v(0) = a(0) = s(180°) = v(180°) = a(180°) = 0$$

and $s(90°) = h$. With 7 BC's we need 7 coefficients. If we solve as we have previously on pp. 5-2, the function turns out to be a 3-4-5-6 polynomial

$$s = h\left[64\left(\frac{\theta}{\beta}\right)^3 - 192\left(\frac{\theta}{\beta}\right)^4 + 192\left(\frac{\theta}{\beta}\right)^5 - 64\left(\frac{\theta}{\beta}\right)^6\right]$$

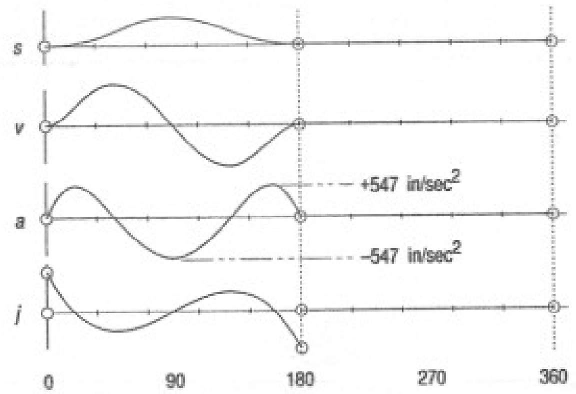

FIGURE 8-30
3-4-5-6 polynomial function for two-segment symmetrical rise-fall, single-dwell cam

Note that the acceleration is reduced compared to the double harmonic above but the jerk is discontinuous at the ends (0 & 180°)

- We could have also set $j(0) = j(180°) = 0$ but that would increase the order of the polynomial by two.